Spherical shock waves in steep density gradients of expanding media

Taya Govreen-Segal

with Ehud Nakar and Amir Levinson

February 24, 2022

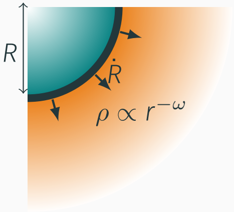
Tel Aviv University

The strong explosion problem

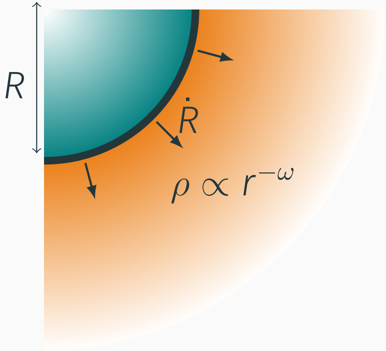
- Energy is released in the center of a spherical density profile
- For power-law density profiles, there is a class of self-similar solutions:
    - Decelerating shocks (Sedov 1946; Von Neumann 1947; Taylor 1950; Blandford and McKee 1976)

    $\Rightarrow$ The energy causally connected to the shock is a conserved quantity
    - Accelerating shocks (Waxman and Shvarts 1993; Sari 2006)

    $\Rightarrow$ The energy causally connected to the shock decreases as the shock accelerates

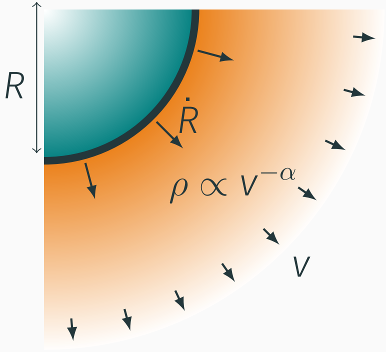


What if we take the density profile



What if we take the density profile

and let it expand?



Motivation

Where might we find a shock in expanding media?

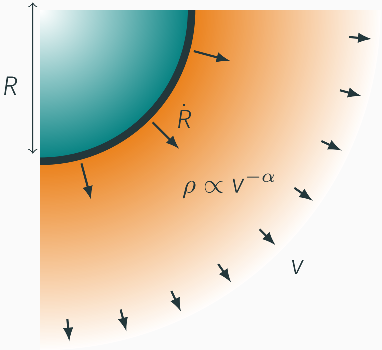
- Possibly in various types of **Supernovae** - A central engine may drive an additional shock into SN ejecta.
  - For example, A magnatar wind in a Supernovae ejecta (Kasen and Bildsten 2010).
- In a **Neutron star merger**, mass is expelled, and the central engine can drive a wide wind creating a quasi-spherical shock.

Existing solutions

- Solutions in static media
- In expanding media - shocks driven by a continuous wind in a shallow density profile (Chevalier 1984; Jun 1998; Suzuki and Maeda 2017).

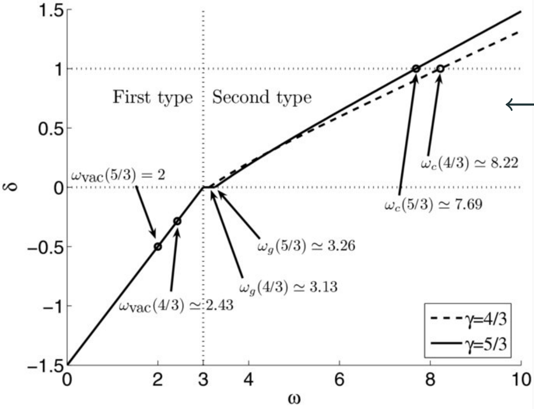
Setup

- A homologously expanding ejecta $v = \frac{r}{t}$ made up of ideal gas
- with a density profile $\rho \propto t^{-3} v^{-\alpha}$, with $\alpha > 5$, so that most of the energy is carried by slower matter (e.g; In SN ejecta, $\alpha \simeq 9 - 12$)
- A Newtonian shock is forced into the expanding media by a strong explosion
- In many cases, also applicable to a shock driven by a fast wind.



Shock propagation in static medium

- A shock forced into a density profile $\rho \propto r^{-\omega}$
- Asymptotically, solutions are of the form $\dot{R} \propto R^{\delta}$

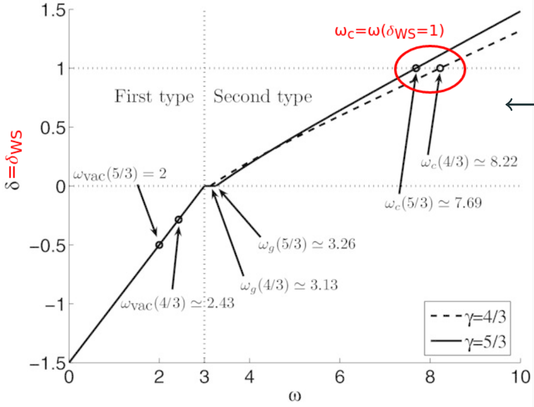


(Kushnir and Waxman 2010)

← Waxman and Shvarts 1993

Shock propagation in static medium

- A shock forced into a density profile $\rho \propto r^{-\omega}$
- Asymptotically, solutions are of the form $\dot{R} \propto R^{\delta}$



Waxman and Shvarts 1993

(Kushnir and Waxman 2010)

Expanding media - properties of the solution

In a static medium, what can I determine from the density power law ω and the shock velocity?

$\Rightarrow$ The full shock evolution (except for normalization)

Expanding media - properties of the solution

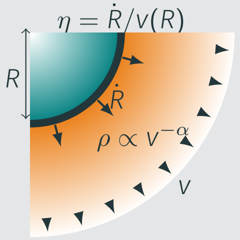
In a static medium, what can I determine from the density power law ω and the shock velocity?

$\Rightarrow$ The full shock evolution (except for normalization)

In an expanding medium, what can I determine from the density power law α and the shock velocity?

$\Rightarrow$ Nothing

Definitions



Expanding media - properties of the solution

In a static medium, what can I determine from the density power law ω and the shock velocity?

$\Rightarrow$ The full shock evolution (except for normalization)

In an expanding medium, what can I determine from the density power law α and the shock velocity?

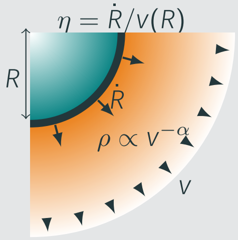
$\Rightarrow$ Nothing

Two velocity scales

The shock velocity and the media velocity right ahead of the shock

$\Rightarrow$ The solution is generally not self-similar.

Definitions

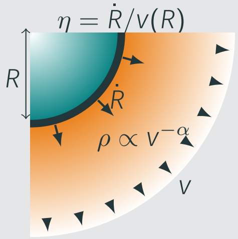


Expanding media - properties of the solution

What determines whether the shock will diverge or die out?

The shock acceleration $\ddot{R}$

Definitions

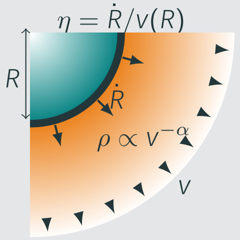


Expanding media - properties of the solution

What determines whether the shock will diverge or die out?

- ✗ The shock acceleration $\ddot{R}$

Definitions



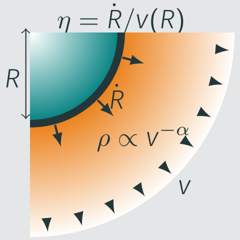
Expanding media - properties of the solution

What determines whether the shock will diverge or die out?

- ✗ The shock acceleration $\ddot{R}$

  The shock acceleration as measured in the immediate upstream $\frac{d}{dt}(\dot{R} - v(R))$

Definitions

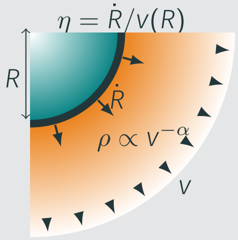


Expanding media - properties of the solution

What determines whether the shock will diverge or die out?

- ✗ The shock acceleration $\ddot{R}$
- The shock acceleration as measured in the immediate upstream $\frac{d}{dt}(\dot{R} - v(R))$
- The shock velocity in units of the velocity of the immediate upstream $\frac{d}{dt}\left(\frac{\dot{R}}{v(R)}\right)$

Definitions

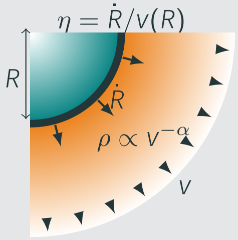


Expanding media - properties of the solution

What determines whether the shock will diverge or die out?

- ✗ The shock acceleration $\ddot{R}$
- ✗ The shock acceleration as measured in the immediate upstream $\frac{d}{dt}(\dot{R} - v(R))$
- ✓ The shock velocity in units of the velocity of the immediate upstream $\frac{d}{dt}\left(\frac{\dot{R}}{v(R)}\right)$

Definitions



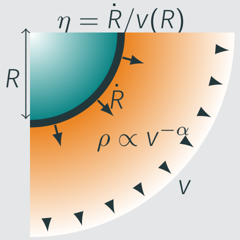
Expanding media - properties of the solution

What determines whether the shock will diverge or die out?

- ✗ The shock acceleration $\ddot{R}$
- ✗ The shock acceleration as measured in the immediate upstream $\frac{d}{dt}(\dot{R} - v(R))$
- ✓ The shock velocity in units of the velocity of the immediate upstream $\frac{d}{dt}\left(\frac{\dot{R}}{v(R)}\right)$

$\Rightarrow$ **The relevant velocity parameter is** $\eta = \frac{\dot{R}}{v(R)}$

Definitions

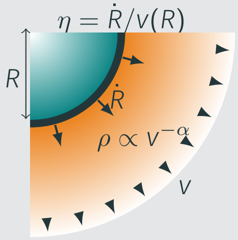


Expanding media - properties of the solution

When do we have a self-similar solution?

- **In the limit of $\eta \to \infty$,** the solution approaches the static case

Definitions

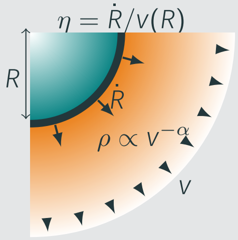


Expanding media - properties of the solution

When do we have a self-similar solution?

- **In the limit of** $\eta \to \infty$, the solution approaches the static case
- **For constant** η; If a solution with constant η exists, than it is self-similar

Definitions

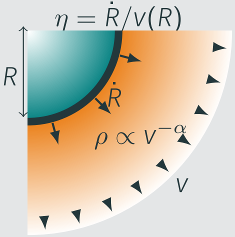


One last definition

$$\delta = \frac{d \log \dot{R}}{d \log R}$$

- In self-similar solutions, δ is a constant of the solution, $\dot{R} \propto R^{\delta}$
- In the static limit, $\eta \gg 1$, $\delta = \delta_{WS}$
- Generally, δ quantifies the shock expansion rate

Definitions



$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

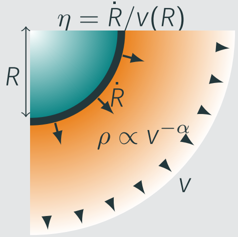
Expanding media - properties of the solution

One last definition

$$\delta = \frac{d \log \dot{R}}{d \log R}$$

- In self-similar solutions, δ is a constant of the solution, $\dot{R} \propto R^{\delta}$
- In the static limit, $\eta \gg 1$, $\delta = \delta_{WS}$
- Generally, δ quantifies the shock expansion rate

Definitions



$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

$$\delta = \frac{d \log \dot{R}}{d \log R}$$

Expanding media - Will the shock grow or decay?

Terming:

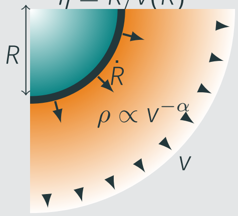
- $\dot{\eta} > 0$ - The shock is ***growing***
- $\dot{\eta} < 0$ - The shock is ***decaying***

I'm going to show that:

- If at some point the shock is *growing*, it will continue to grow and it's radius will diverge
- It at some point the shock is *decaying*, it's velocity will approach a constant and it will die out.

Definitions

$$\eta = \dot{R}/v(R)$$



$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

$$\delta = \frac{d\log\dot{R}}{d\log R}$$

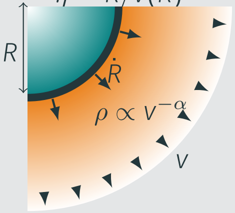
Expanding media - Will the shock grow or decay?

Deriving η according to time:

$$\frac{d\log\eta}{d\log t} = (\delta - 1)\eta + 1.$$

Definitions

$$\eta = \dot{R}/v(R)$$



$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

$$\delta = \frac{d\log\dot{R}}{d\log R}$$

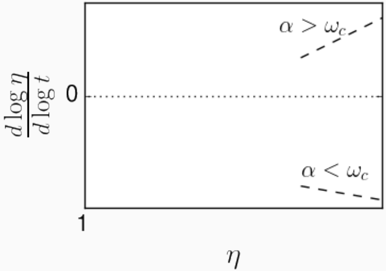
Expanding media - Will the shock grow or decay?

Deriving η according to time:

$$\frac{d\log\eta}{d\log t} = (\delta - 1)\eta + 1.$$

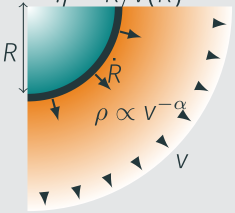
- If $\eta \gg 1$:

$$\frac{d\log\eta}{d\log t} \simeq (\delta_{WS} - 1)\eta \begin{cases} \nearrow +, \delta_{WS} > 1 \\ \searrow -, \delta_{WS} < 1 \end{cases}$$



Definitions

$\eta = \dot{R}/v(R)$



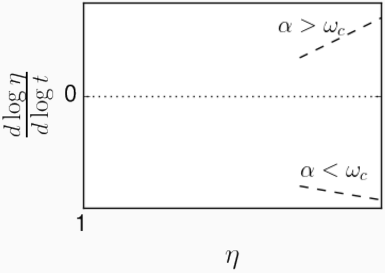
$$\omega_c = \omega\,(\delta_{WS} = 1)$$

$$\delta = \frac{d\log\dot{R}}{d\log R}$$

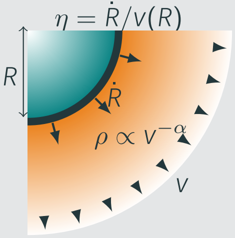
Deriving η according to time:

$$\frac{d\log\eta}{d\log t} = (\delta - 1)\eta + 1.$$

- For $\dot{\eta}$ to change a sign, it must vanish.
- $\dot{\eta} = 0 \iff \eta = \eta_c = \dfrac{1}{1-\delta}$



Definitions

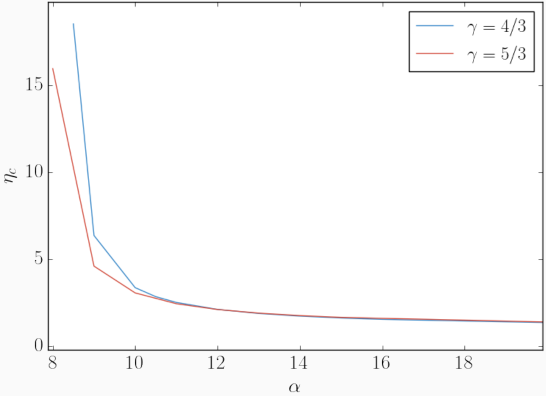


$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

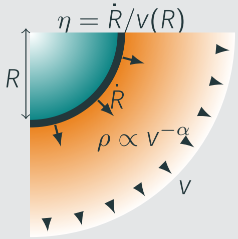
$$\delta = \frac{d\log\dot{R}}{d\log R}$$

The self-similar solution

- For every $\alpha > \omega_c$, a single self-similar solution exists
- For $\alpha \to \omega_c$ (from above), $\eta_c \to \infty$
- For $\alpha < \omega_c$, there is no self-similar solution



Definitions



$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

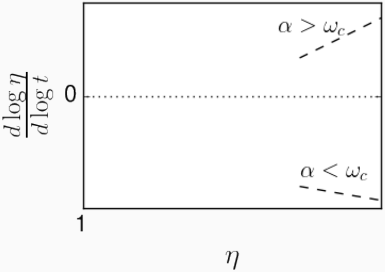
$$\delta = \frac{d \log \dot{R}}{d \log R}$$

Expanding media - Will the shock grow or decay?

Deriving η according to time:

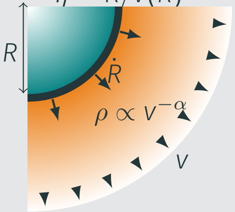
$$\frac{d\log\eta}{d\log t} = (\delta - 1)\eta + 1.$$

- For $\dot{\eta}$ to change a sign, it must vanish. This happens only for $\alpha > \omega_c$
- $\dot{\eta} = 0 \iff \eta = \eta_c = \dfrac{1}{1-\delta}$



Definitions

$\eta = \dot{R}/v(R)$



$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

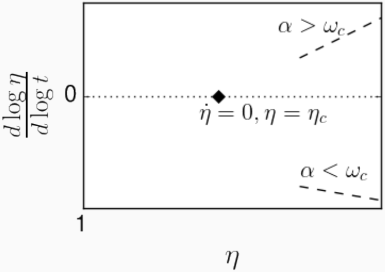
$$\delta = \frac{d\log\dot{R}}{d\log R}$$

Expanding media - Will the shock grow or decay?

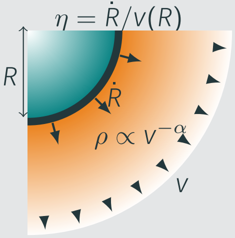
Deriving η according to time:

$$\frac{d \log \eta}{d \log t} = (\delta - 1)\eta + 1.$$

- For $\dot{\eta}$ to change a sign, it must vanish. This happens only for $\alpha > \omega_c$
- $\dot{\eta} = 0 \iff \eta = \eta_c = \dfrac{1}{1-\delta}$



Definitions



$$\omega_c = \omega\,(\delta_{WS} = 1)$$

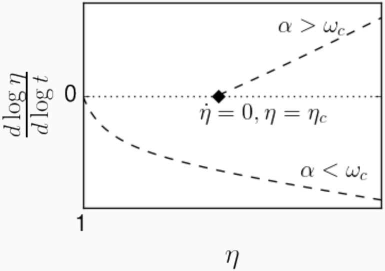
$$\delta = \frac{d \log \dot{R}}{d \log R}$$

Expanding media - Will the shock grow or decay?

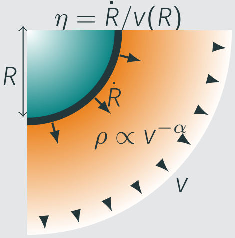
Deriving η according to time:

$$\frac{d\log\eta}{d\log t} = (\delta - 1)\eta + 1.$$

- For $\dot{\eta}$ to change a sign, it must vanish. This happens only for $\alpha > \omega_c$
- $\dot{\eta} = 0 \iff \eta = \eta_c = \dfrac{1}{1-\delta}$



Definitions



$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

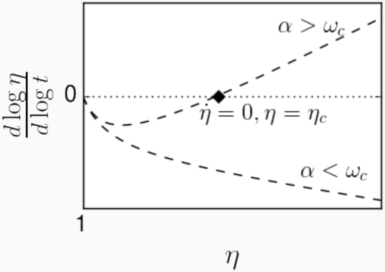
$$\delta = \frac{d\log\dot{R}}{d\log R}$$

Expanding media - Will the shock grow or decay?

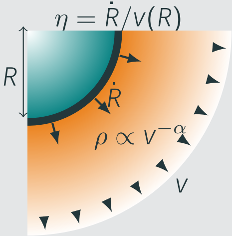
Deriving η according to time:

$$\frac{d\log\eta}{d\log t} = (\delta - 1)\eta + 1.$$

- For $\dot{\eta}$ to change a sign, it must vanish. This happens only for $\alpha > \omega_c$
- $\dot{\eta} = 0 \iff \eta = \eta_c = \dfrac{1}{1-\delta}$



Definitions



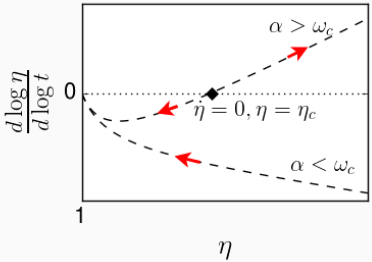
$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

$$\delta = \frac{d\log\dot{R}}{d\log R}$$

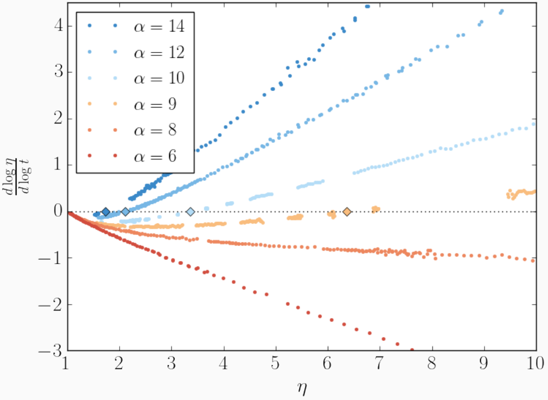
Summary so far

$\alpha < \omega_c$	$\alpha \geq \omega_c$	
shock decays ($\dot{\eta} < 0$)	$\eta < \eta_c$	shock decays ($\dot{\eta} < 0$)
	$\eta = \eta_c$	self-similar solution
	$\eta > \eta_c$	shock grows ($\dot{\eta} > 0$)

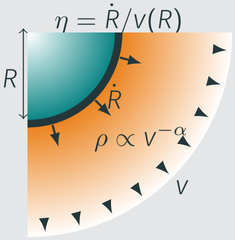
- $\omega_c \simeq 8.22$ for $\gamma = \frac{4}{3}$
- $\omega_c \simeq 7.69$ for $\gamma = \frac{5}{3}$



- The self-similar solution is repelling



Definitions

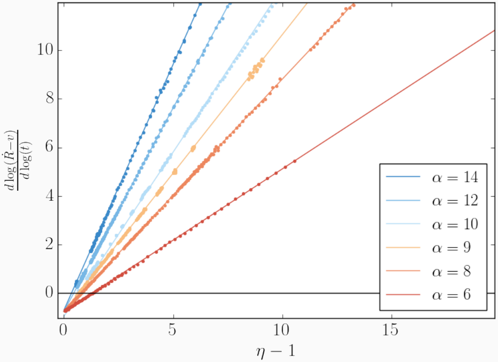


$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

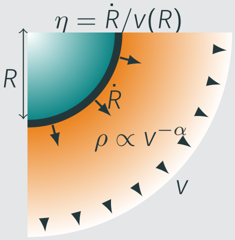
$$\delta = \frac{d \log \dot{R}}{d \log R}$$

Expanding media - Analytic Description

- Assuming the relation in the graph is linear, there exists a full analytic description



Definitions



$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

$$\delta = \frac{d\log \dot{R}}{d\log R}$$

Assuming $\frac{d\log(\dot{R}-v)}{d\log(t)} = a\cdot(\eta-1)+b$, we find that:

- $a=\delta_{WS}$ and $b=(\eta_c-1)(\delta_{WS}-1)$

The shock location:

$$\frac{R}{R_i} = \begin{cases} \frac{t}{t_i}\left(\frac{b}{b+\left(\frac{t}{t_i}\right)^{-b}(\delta_{WS}-1)(\eta_i-1)\left(1-\left(\frac{t}{t_i}\right)^{-b}\right)}\right)^{\frac{1}{\delta_{WS}-1}}, & \delta_{WS}\neq 1 \\ \frac{t}{t_i}\exp\left(\frac{\left(1-\left(\frac{t}{t_i}\right)^{-b}\right)(\eta_i-1)}{b}\right), & \delta_{WS}=1 \end{cases}$$

Expanding media - Analytic Description

Assuming $\frac{d\log(\dot{R}-v)}{d\log(t)} = a\cdot(\eta-1)+b$, we find that:

- $a=\delta_{WS}$ and $b=(\eta_c-1)(\delta_{WS}-1)=\begin{cases}0.75, & \gamma=4/3\\ 1, & \gamma=5/3\end{cases}$

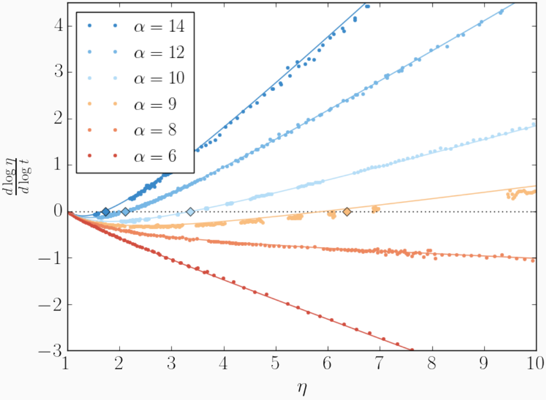
The divergence time (for growing shocks):

$$t(R\to\infty)=t_i\left(\frac{\eta_i-1}{\eta_i-\eta_c}\right)^{\frac{1}{b}} \tag{1}$$

And the die-out velocity (for dissipating shock)s:

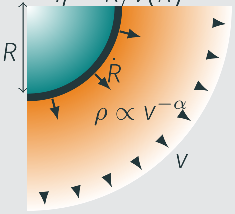
$$\lim_{t\to\infty}\dot{R}=\frac{R_i}{t_i}\begin{cases}\left(\frac{b}{b-(\delta_{\mathrm{WS}}-1)(\eta_i-1)}\right)^{\frac{1}{\delta_{WS}-1}}, & \delta_{WS}\neq 1\\ \exp\left(\frac{\eta_i-1}{b}\right), & \delta_{WS}=1\end{cases} \tag{2}$$

Expanding media - Analytic Description



Definitions

$\eta = \dot{R}/v(R)$



$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

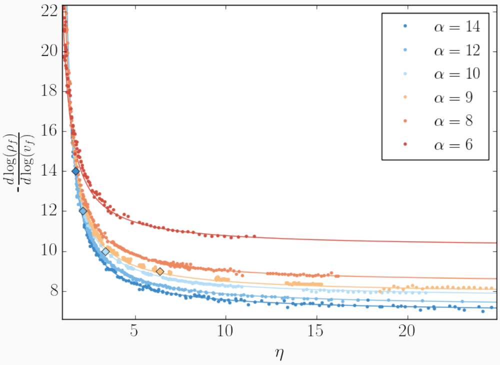
$$\delta = \frac{d \log \dot{R}}{d \log R}$$

Observational Signature (Not Really)

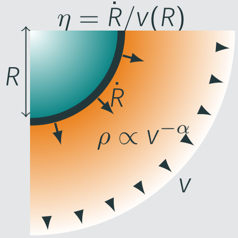
Expanding media - Density profile far behind the shock

Defining $\alpha_f = -\dfrac{d \log \rho_f}{d \log v_f}$:

decaying shocks	self-similar solution	growing shocks	$\eta \gg 1$
$\alpha_f > \alpha$	$\alpha_f = \alpha$	$\alpha_f < \alpha$	$\alpha_f = \dfrac{\alpha}{\delta_{WS}} \simeq 8 - 10$

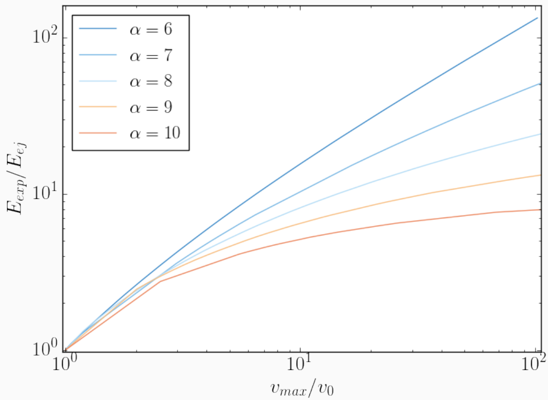


Definitions

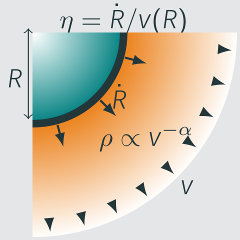


Expanding media - Breakout from finite media

- A shock enters the steep part of the ejecta at v_0. How much energy does it need to reach v_{max}?
- For supernovae ejecta, $\frac{v_{max}}{v_0} \simeq 5 - 20$



Definitions



$$\omega_c = \omega\left(\delta_{WS} = 1\right)$$

$$\delta = \frac{d \log \dot{R}}{d \log R}$$

Summary

- Shocks with $\rho \propto v^{-\alpha}$, $\alpha < \omega_c$ always decay ($\omega_c \simeq 8$)
- Shocks with $\alpha > \omega_c$ need a large enough initial velocity in order not to decay
- The density profile behind the shock is altered, becoming steeper if the shock decays, and milder if the shock grows
- Decaying shocks can still break out is they have enough energy ($E_{exp} \sim 10E_{ej}$)

In the paper (arXiv:2010.10543 [astro-ph.HE])

- Derivation of the self-similar solution
- Analytical description of the shock propagation
- Derivation of the density profile far behind the shock and of the energy required for a shock breakout
- Discussion of the applicability to a shock driven by a fast wind

References

Blandford, R. D. and C. F. McKee (Aug. 1976). “Fluid dynamics of relativistic blast waves”. In: *Physics of Fluids* 19, pp. 1130–1138. DOI: 10.1063/1.861619.

Chevalier, R. A. (May 1984). “The interaction of Crab-like supernova remnants with their surroundings.”. In: 280, pp. 797–801. DOI: 10.1086/162053.

Jun, Byung-Il (May 1998). “Interaction of a Pulsar Wind with the Expanding Supernova Remnant”. In: 499.1, pp. 282–293. DOI: 10.1086/305627. arXiv: astro-ph/9712054 [astro-ph].

References ii

Kasen, Daniel and Lars Bildsten (July 2010). "Supernova Light Curves Powered by Young Magnetars". In: 717.1, pp. 245–249. DOI: 10.1088/0004-637X/717/1/245. arXiv: 0911.0680 [astro-ph.HE].

Kushnir, Doron and Eli Waxman (2010). "Closing the gap in the solutions of the strong explosion problem: An expansion of the family of second-type self-similar solutions". In: *Astrophys. J.* 723.1, pp. 10–19. ISSN: 15384357. DOI: 10.1088/0004-637X/723/1/10.

Sari, Re'em (Feb. 2006). "First and second type self-similar solutions of implosions and explosions containing ultrarelativistic shocks". In: *Physics of Fluids* 18.2, pp. 027106–027106. DOI: 10.1063/1.2174567. arXiv: astro-ph/0505174 [astro-ph].

- Sedov, Leonid I (1946). "Propagation of strong shock waves". In: *J. Appl. Math. Mech.* 10, pp. 241–250.
- Suzuki, Akihiro and Keiichi Maeda (Apr. 2017). "Supernova ejecta with a relativistic wind from a central compact object: a unified picture for extraordinary supernovae". In: 466.3, pp. 2633–2657. DOI: `10.1093/mnras/stw3259`. arXiv: `1612.03911 [astro-ph.HE]`.
- Taylor, Geoffrey (Mar. 1950). "The Formation of a Blast Wave by a Very Intense Explosion. I. Theoretical Discussion". In: *Proceedings of the Royal Society of London Series A* 201.1065, pp. 159–174. DOI: `10.1098/rspa.1950.0049`.
- Von Neumann, J (1947). "Blast Waves (Los Alamos Sci. Lab. Tech. Series), Vol.". In: *LI Sedov (Los Alamos, NM)*.

Waxman, Eli and Dov Shvarts (Apr. 1993). "Second-type self-similar solutions to the strong explosion problem". In: *Physics of Fluids A* 5.4, pp. 1035–1046. DOI: 10.1063/1.858668.