3D Gaussian Beam Tracing

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Introduction

- Since Dec. 2015: three GW signals [1, 2, 3]
- Assessment of gravitational astronomy as a viable component of multi-messenger astronomy
- Today: final steps of the Advanced Virgo interferometer commissioning
- Particularities of optics at gravitational observatories ⇒ New optics tool

- **What is peculiar about optics at GO?**
- **What physics are at hand?**
- **The `theia` tool**

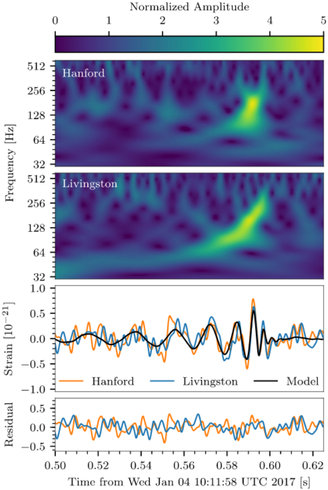

Figure: Spectrogram and GW170104 signal (from [3])

Overview

1. Quick return on GW
2. Optics at Virgo
3. The physics of general astigmatic Gaussian beams
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Return on GW

The GO network

Grav. obs.:

- Multi-directional
- Multi-spectral (broad-band)
- Orthogonal to traditional channels (EM, particles)

Instrument	**Beginning of service**	**Frequency range** (Hz)	**Sensitivity** (100 Hz)	**Features**
Advanced Virgo	2017	10 – 10 000	$10^{-23.7}$	3 km arms
aLIGO	Nov. 2016 (O2 run start)	id.	$10^{-23.8}$ (BNS-optimized)	4 km arms
KAGRA	∼ 2019 (pro.)	sim.	sim.	Under-ground, 3 km arms, cryogenized (20 K) sapphire test masses
Einstein Telescope	2025?	10 – 10 000	$10^{-24.5}$	Under-ground, 10 km arms, triangular topology
LISA	2034	$10^{-3} - 1$	10^{-22}	Space-bound, $\sim 10^6$ km arms, triangular topology

Table: The future gravitational interferometry network (source: [5, 4, 6, 7]).

Return to GW

3 first detections

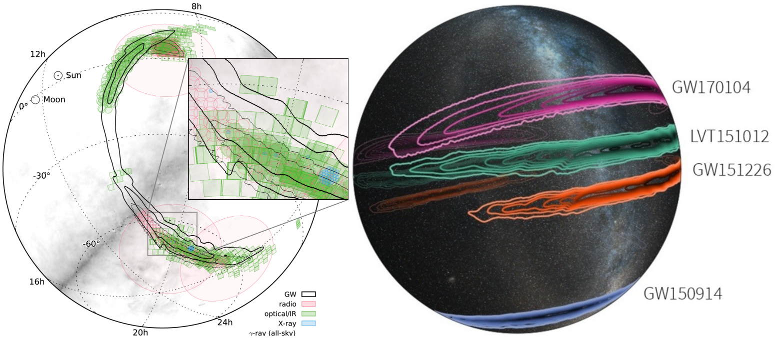

Figure: *Left*: EM coincident events with to GW150914 [8], *right*: sky map of three first detections (90% confidence)

Historical moment

- Culminates ∼ 50 years of research
- Entry of GA in the landscape of astronomy
- Localization: ∼ 1000 deg^2 ∼ 2.5% of the sky
- Follow-up by EM astronomy in network after 4th detection [6]

Return to GW

The propagation of light in a GW environment

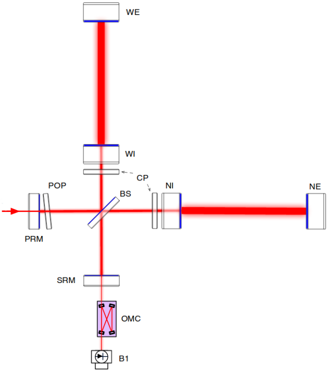

Figure: The Advanced Virgo setup

- Michelson signal: $\Delta P(t) = -2P_0 k \sin(2k(a_0 - b_0))x(t)$, $x(t)$: difference of lengths of arms.
- $$h^{TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+(t,z) & h_\times(t,z) & 0 \\ 0 & h_\times(t,z) & -h_+(t,z) & \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
- Light null geodesic along the x direction: $0 = c^2dt^2 - dx^2 + h_+(t)dx^2 \;\leftrightarrow\; dx = \pm cdt\left(1 + \frac{h_+}{2}\right)$
- For a round trip from $x(t_0) = 0$ to $x(t_1) = a_0$ and back at t_2:
$$2L = c(t_2 - t_0) + \frac{1}{2}c\int_{t_0}^{t_2} h_+(u)du$$
- Implicit equation in t_2 solved at first order supposing $a_0, b_0 \ll \lambda_{\rm GW} \Leftrightarrow \Omega_{\rm GW} \ll 10^5$ Hz:
$$x(t) = \Delta a(t) - \Delta b(t) = h_+(t)\frac{a_0 + b_0}{2}$$
- 3rd detection: $x(t) \sim 10^{-18}$ m

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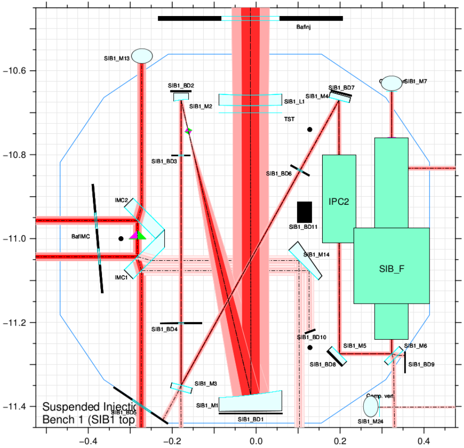
Optics at Virgo

Optical constraints



Physical constraints

- Kilometer-scale setup with 12 benches → general overview → CAD export
- Stray light (scattered light and ghost beams) tolerance $\sim 10^{-21}$ in FP → beam hunting capabilities
- Wedges and non-planarity → 3D optics
- Configuration exploration and effective optics → scripting

Computational constraints

- $\sim$ 500 optics and a typical simulation is $\sim$ 700 beams
- Adapted data structure
- Documentation and clear sources

Optics at Virgo

3D setups?

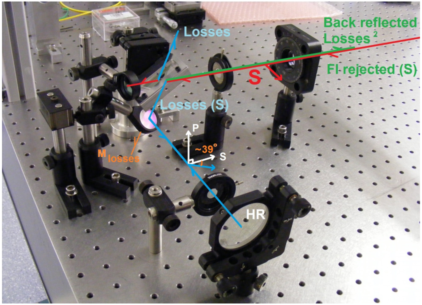


Figure: A periscope in the optics lab at Virgo (Optics logbook entry 1029, courtesy of G. Pillant)

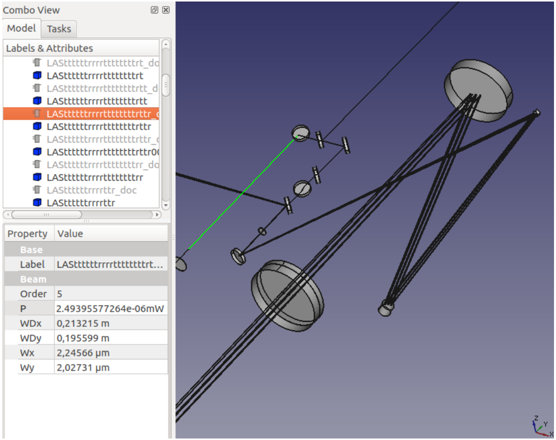
What effects do 3D setups have?

- Polarization shifts
- Interference complications because of general astigmatism

Can we predict the behavior of the beam? ⇒ General astigmatic Gaussian beams

Optics at Virgo

Summary: What do we need?



LASttttttrrrrttttttttrt_do
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LASttttttrrrrttttttttrr_do
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Property	Value
Base	
Label	LASttttttrrrrttttttttrt...
Beam	
Order	5
P	2.49395577264e-06mW
WDx	0,213215 m
WDy	0,195599 m
Wx	2,24566 µm
Wy	2,02731 µm

Figure: The expected work flow of new software

Physics:

- 3D general setups (optics position and orientation, cavities)
- General astigmatic Gaussian beams
- Interferences

Features:

- Scripting capabilities
- 3D navigation into the optics setup
- Navigation in the beam tree for stray light hunting
- High level text input/output

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General astigmatic Gaussian beams

From orthogonal to astigmatic beams

- Appears on oblique interactions (elliptical optical components)
- Characteristics: No orthonormal basis $\Leftrightarrow$ phase and power ellipses shift
- Most general solution of the eikonal equation $(\Delta_{x,y} + 2ik\partial_z)E(x,y,z) = 0$ within the paraxial approximation $\partial_z E \ll kE$:

$$E(x,y,z) = E_0(z)\exp\left[i\eta(z) - i\frac{k}{2}{}^t rQ(z)r\right]$$

with $Q(z)$ the curvature tensor $(q_{x,y}(z) = z - z^0_{x,y} + iz^R_{x,y},\ \theta \in \mathbb{C})$:

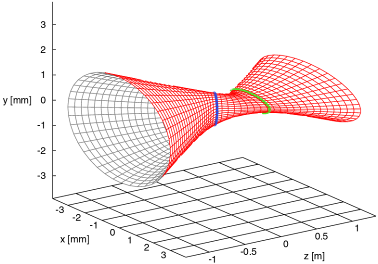

Figure: The constant amplitude $|E(x,y,z)|^2$ surface of an astigmatic beam (from [9])

$$Q(z) = \begin{pmatrix} \frac{\cos^2\theta}{q_x(z)} + \frac{\sin^2\theta}{q_y(z)} & \frac{1}{2}\sin 2\theta\left(\frac{1}{q_x(z)} - \frac{1}{q_y(z)}\right) \\ \frac{1}{2}\sin 2\theta\left(\frac{1}{q_x(z)} - \frac{1}{q_y(z)}\right) & \frac{\sin^2\theta}{q_x(z)} + \frac{\cos^2\theta}{q_y(z)} \end{pmatrix}$$

General astigmatic Gaussian beams

How do these beams transform? The phase-matching method [9]

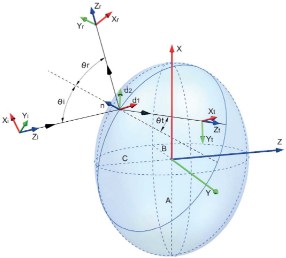

Figure: The geometry of an incidence on a spherical surface (from [9])

Approximations

- Geometrical optics: none
- Gaussian data: ROC(beam) ≫ ROC(surface) (+ paraxial)

Question Incident beam on spherical surface Q_i, transmitted and reflected Q_t, Q_r?

- Orthonormal coordinates: (x_l, y_l, z_l) for each beam $l = i, t, r$ and matrix
- Phase of each beam: $\Phi_l(r_l) = \frac{1}{2}{}^t r_l Q_l r_l$
- Hypothesis: *Phases of all beams coincide on the interaction surface*. Thus, $\forall l\ \forall r_l\ \Phi_l(r_l) = \Phi_i(r_i)$
- Expressing the r_l in the basis normal to the sphere and using the definition of the sphere leads to the final law:

$$Q_r = {}^tK_r^{-1}({}^tK_iQ_iK_i - \frac{1}{R}(n \cdot z_i - n \cdot z_r))K_r^{-1}$$

and

$$Q_t = \frac{n_1}{n_2}{}^tK_t^{-1}({}^tK_iQ_iK_i - \frac{1}{R}(n \cdot z_i - \frac{n_2}{n_1}n \cdot z_r))K_r^{-1}$$

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The `theia` tool

Scope of theia

No	**Yes** (July 1^{th})	**Not yet**	**Extensions**
Higher order modes	3D general astigmatic Gaussian beams	All 2^{nd} surfaces	Polarization
Grating surfaces	Spherical surfaces (mirrors, lenses)	High-level 3D visualization	Surface action specification
Response in GW environment	{Non-, }sequential tracing	Cavities	2-way communication with CAD
	Low-level 3D visualization	Interferences	
		Beam tree navigation interface	

Table: The functionalities of `theia`

The `theia` tool

Operation of `theia`

Algorithms + Data Structures = Programs

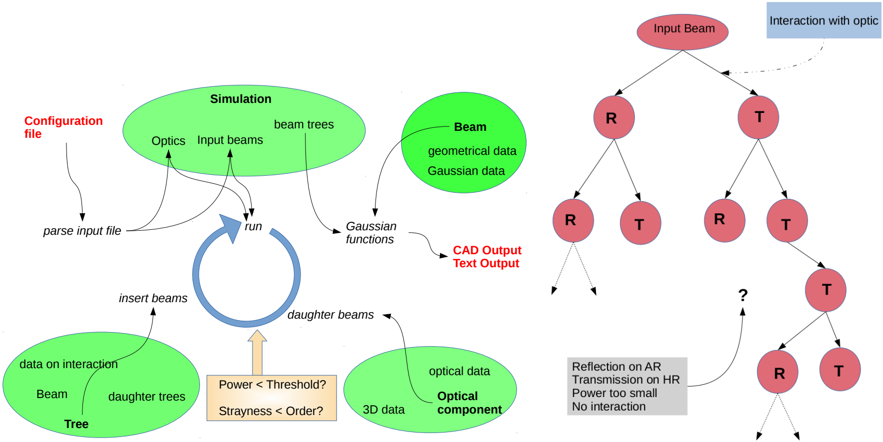

Figure: *Left*: the algorithm implemented in `theia`, *right*: the beam tree data structure.

Test Cases

Optical bench design

- Optimum placement of lens *L*2 for maximum width of transmitted beam? for distance of waist?
  ⇒ Small script

Figure: Optical setup (*left*) and Python script (*right*) for an optimization case

```
#simulation object
simu = simulation.Simulation(FName = 'optimization')
simu.LName = 'Optimizing with theia!'
simu.Order = 0
simu.Threshold = 0.5*mW
```

Setup

```
#optics, the first L1 lens doesn't move
L1 = thinlens.ThinLens(X = 0*cm, Y = 0., Z = 0., Focal = 20.*cm,
            Diameter = 3.*cm, Phi = 180.*deg, Ref = 'L1')

bm = beam.userGaussianBeam(1.*mm, 1.*mm, 0., 0, 1064*nm, 1*W,
                            X = -30*cm, Phi = 0, Ref = 'Beam')

# this is a list of centers for the second lens we want to try (it is around
#70 cm = L1.X + 2*Focal to respect the 2F configuration). We're trying n
#configurations around L2.X = 50
n = 500
centers = [ 40.*cm + 2.*cm*(float(i)/n) for i in range(-n, n)]
waistSizes = []
waistPositions = []

# load beam
simu.InBeams = [bm]
```

Optical data specification

```
#run the simulations in sequence!
for center in centers:
    dic = {'X': center, 'Y': 0., 'Z': 0., 'Focal': 20.*cm,
        'Diameter': 3.*cm, 'Theta': 90*deg, 'Phi': 180.*deg, 'Ref': 'L2'}
    simu.OptList = [L1, thinlens.ThinLens(**dic)]

    #go theia go!
    simu.run()
    output = simu.BeamTreeList[0].T.T.T.T.Root

    #save output data for plotting
    waistSizes.append(output.waistSize()[0])
    waistPositions.append(output.waistPos()[0])

simu.writeCAD()
```

Optimization sequence

```
#plot the results
plt.figure(1)
plt.subplot(211)
plt.plot(centers, waistSizes, 'r')
plt.ylabel('waists (m)')
plt.xlabel('center of second lens (m)')
plt.subplot(212)
plt.plot(centers, waistPositions, 'g')
plt.ylabel('positions (m)')
plt.xlabel('center of second lens (m)')
plt.show()
```

Plotting

Test Cases

Optical bench design

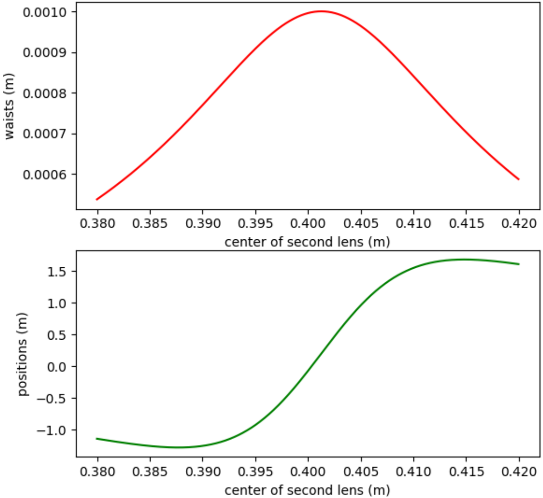

Figure: Width (*top*) and position (*bottom*) of the waist of the output beam as calculated by `theia`

Conclusions and perspectives

- New simulation tool to respond to requirements of gravitational astronomy
- First test cases reveal scripting and simulation potential
- Next step: **confirm test cases, implement following features, communicate** (`theia.hopto.org:56000`)

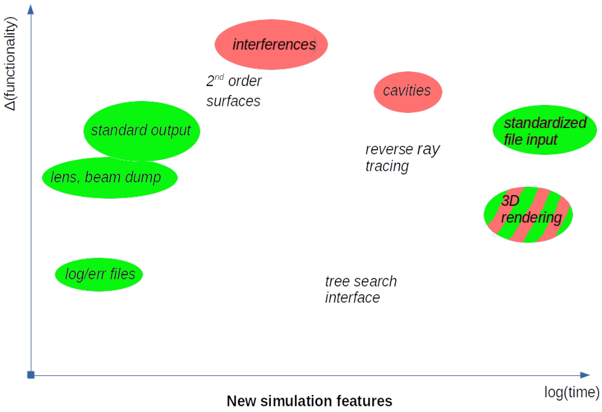

Figure: The near future of `theia`

References

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[2] LIGO Scientific Collaboration and Virgo Collaboration: *GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence*, Phys. Rev. Lett. 116 (2016)

[3] LIGO Scientific Collaboration and Virgo Collaboaration: *GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2*, Phys. Rev. Lett. 118 (2017)

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[6] LIGO Scientific Collaboration and Virgo Collaboration: *Prospects for Observing and Characterizing GW Transients with aLIGO and Advanced Virgo*, arXiv: 1304.0670 (2016)

[7] The KAGRA Collaboration: *Interferometer Design of the KAGRA GW Detector*, arXiv:1306.6747 (2013)

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[9] E. Kochkina, G. Wanner, D. Schmelzer, M. Tröbs, G. Heinzel: *Modeling of the General Astigmatic Gaussian Beam and its Propagation through 3D Optical Systems*, Applied Optics 24 (2013)

Return on GW

The reach of gravitational astronomy

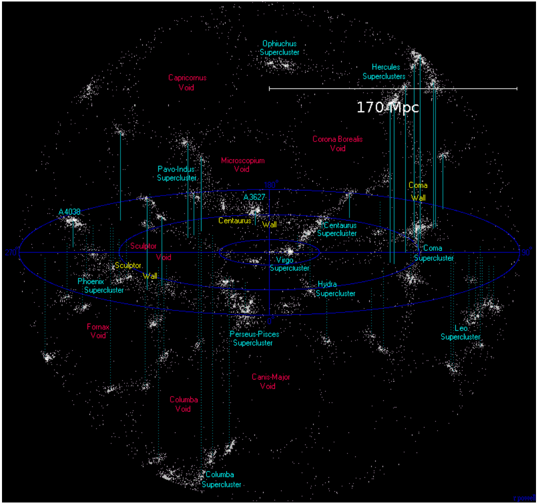


Figure: The local Universe in the reach of aLIGO and AdV (adapted from R. Powell, Wikicommons)

- Sensitivity of aLIGO/AdV: $\leq 10^{-23.5}$ $\mathrm{Hz}^{-1/2}$ on 50–1000Hz ([6])
- BNS ranges (1,4 $M_\odot$, SNR$\geq$8): aLIGO 200 Mpc, AdV 130 Mpc
- Expected rates: 0.4–400 events y^{-1} above 8 SNR
- Distances of 3 first signals:
  GW150914, GW151226:
  440 ± 180 Mpc
  GW170104: 880 ± 450 Mpc

Modeling Gaussian beams

- General astigmatic Gaussian beam in an orthogonal basis (k, e_1, e_2):

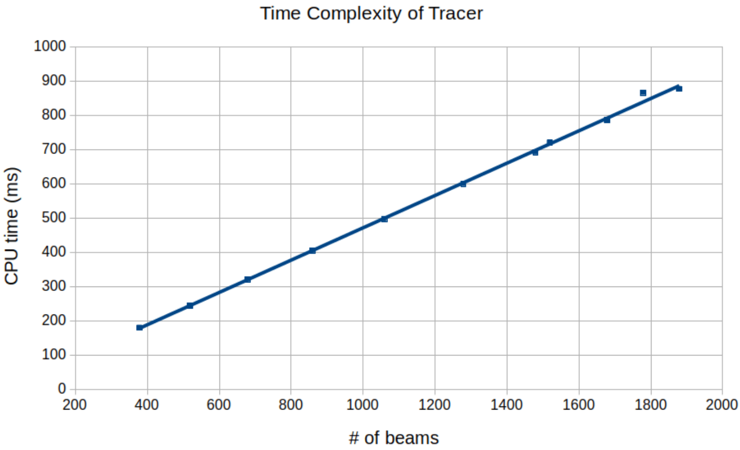
$$E(\vec{r}, t) = \exp[i\eta(z) - i\frac{k}{2}{}^t(x, y)Q(z)(x, y)]e^{i(\omega t - kz)}$$

- (x, y) is the transversal coordinate in the (e_1, e_2) basis, Q is a symmetrical tensor:

$$\begin{pmatrix} \frac{\cos^2\theta}{q_x(z)} + \frac{\sin^2\theta}{q_y(z)} & \frac{1}{2}\sin 2\theta\left(\frac{1}{q_x(z)} - \frac{1}{q_y(z)}\right) \\ \frac{1}{2}\sin 2\theta\left(\frac{1}{q_x(z)} - \frac{1}{q_y(z)}\right) & \frac{\sin^2\theta}{q_x(z)} + \frac{\cos^2\theta}{q_y(z)} \end{pmatrix}$$

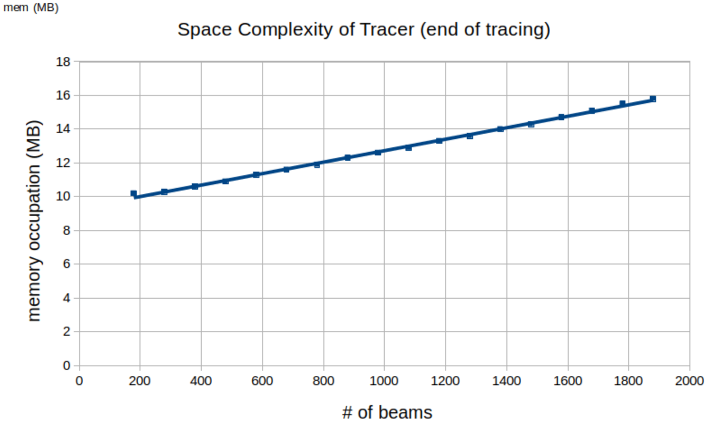
- Specification parameters: $\theta, q_{x,y} \in \mathbb{C}$, (e_1, e_2) basis.
- **Approximations:** ROC(beam) $\gg$ ROC(surface) (+ paraxial)
- Geometric optics: no approximation

Benchmarking: time (i7/8GB)



- CPU = 0.47ms × (# beams) ($R^2 = 99.95\%$)

Benchmarking: space (i7/8GB)



- Mem. $= 9{,}3\text{MB} + 3{,}4\text{kB/beam}$ ($R^2 = 99.76\%$)